3. PRICE DYNAMICS FOR DURABLE GOODS

3.1 Introduction

Expenditure on durable goods accounts for 60% of consumption expenditure and all of investment expenditure. It is the most volatile component of GDP at business cycle frequencies. A large fraction of international trade is in durable goods. As is well known, the durable nature of the product makes the pricing of these goods differ from that of nondurables, since consumers demand for durables today depends not only on prices today but also on their expectation of future prices. In the vast majority of macroeconomic models (both closed and open-economy) that study the behavior of durables pricing of durables is treated similar to that of nondurables: either perfect competition is assumed and firms are price takers or if the firm has pricing power they are assumed to not internalize the effect of their price today on future demand for their product, despite the durable nature of their product (Barsky, House, and Kimball (2007)). In the open economy literature the impact of exchange rate movements on prices is studied in environments where goods are assumed to be nondurable, despite the preponderance of durable goods in trade.

On the other hand there is a large microeconomic literature that studies the specific problem of pricing of durable goods and its special features relative to nondurable pricing. The forward looking nature of demand implies that the prices firms set will depend on whether they can commit to future prices or cannot. In one of the
earliest papers in the literature Coase (1972) conjectured that in the absence of commitment a monopolistic firm producing durable goods will be bound to charge the marginal cost due to perfect inter-temporal competition with itself. This conjecture has been proven by Stokey (1981), Bulow (1982) and Gul, Sonnenschein, and Wilson (1986) in various setups, as discussed in the Supplementary Section 1.5 of Chapter 1 of Tirole (1988). This conjecture is a limiting result in an environment where prices adjust at each instant (continuous time) or there is zero depreciation of the durable good. A large literature has followed as surveyed in Waldman (2003). Much of this analysis has focused on long-run pricing behavior with less emphasis on dynamics and response to shocks. Also, the analysis is typically done for the case of a monopolist. As Waldman (2003) mentions in his conclusion (see p. 150) “most of the literature assumes either monopoly or perfect competition, while clearly most real world markets are either oligopolistic or monopolistically competitive”.

In this work we bring the insights in the microeconomic literature on durable goods pricing into macroeconomic environments. Consistent with macroeconomic treatment of durable goods we allow for positive depreciation rates and discrete time periods between price setting and evaluate several pricing environments including monopoly, oligopoly and monopolistic competition. We explore the cases of commitment and discretion and evaluate the response to cost and demand shocks.

We consider a partial equilibrium environment and focus on firms price setting given a consumer demand function that depends on its prices today and in the future. In the case with commitment, prices are independent of the past levels of durable goods consumption and of past prices. It depends on the current level of demand and on current and future costs. If the elasticity of demand (with respect to the “rental” price of the durable good) is constant then prices are a constant markup over
marginal costs. There are no endogenous dynamics in prices. There is complete and instantaneous pass-through of cost (exchange rate shocks) into prices and demand shocks have no effect on prices. When firms have the ability to commit they are able to commit not to compete with themselves and thus obtain monopoly rents.

As is well known the commitment solution is not dynamically consistent. The demand for the durable good depends on its expected future price. In the current period the monopolist would benefit if the consumers believed that the future prices of the durable good would be high, but in the next period, the monopolist would like to lower the price in order to increase sales. In the absence of the monopolist’s ability to commit to high future prices, the consumers will base their current purchases on their expectation of low future prices. This impedes the producer’s ability to capture the full potential monopoly rent.

We evaluate the implications of the time consistent solution for the dynamics of adjustment to cost and demand shocks and focus on Markov perfect equilibria. In this environment prices depends on the endogenous state variable, the stock of durables. Consequently prices adjust sluggishly to shocks even when the shock is a permanent shock. Markups move endogenously over time. In response to a positive cost shock (exchange rate shock) prices increase but by less than the percent increase in costs and markups decline generating incomplete pass-through. When costs increase firms mute the price response to prevent consumers from shifting demand to the future when they cannot commit to keep prices high (relative to their marginal cost). Demand shocks also now impact pricing. Markups and prices increase in response to positive demand shocks.

The fact that markups decrease in response to a positive cost shock has implications for the literature on incomplete exchange rate pass-through. Most traded goods
are durable in nature. The fact that pass-through is incomplete in the long-run is frequently attributed to strategic complementarities in pricing that prevents a firm from raising its price in response to cost shocks as this causes the elasticity of demand it faces to rise. Adding the assumption of frictions in price adjustment then generates dynamics in pass-through.

Therefore, in the case of durable goods, with discretion in pricing one obtains pass-through dynamics even in the flexible price case, and incomplete pass-through even in the absence of standard strategic complementarities in pricing and constant elasticity of demand. This contrasts with the literature on exchange rate pass-through that treats all goods as nondurable and where endogenous dynamics in prices arise because of infrequent price adjustments and pass-through is incomplete in the long-run because of variable markups that arise from strategic complementarities in pricing. The solution here also contrasts with the standard macroeconomic assumption of marginal cost pricing of durable goods. The endogeneity of markups implies that, in response to cost shocks, the volatility in prices and therefore quantity is lower than the case of constant markup pricing.

In the case of oligopolistic competition where firms engage in Cournot competition in producing a homogenous good the pricing decision of each firm is influenced by two different forces: competition with the other firms and inter-temporal competition with itself. We show here that the extent of dynamics in prices, in the time-consistent solution, is a decreasing function of the number of firms. That is, with a large number of competing firms, the across-firm competition dominates the inter-temporal within firm competition of the firm. In the case of monopolistic competition where each firm produces an individual variety and is assumed to be infinitesimally small relative to the industry we show that the problem is similar that of a durable
good monopolist subject to stochastic demand shocks. We also show that the price of a particular variety (and the persistence in price dynamics) of durable good depends more strongly on the deviation of the average stock of durables in the industry from its steady state as compared to the deviations of the stock of its variety relative to the industry average.

In future versions of this work we plan to nest the durable goods pricing problem into a general equilibrium macroeconomic environment. A paper in the literature that speaks to the dynamic response to demand shocks is Caplin and Leahy (2006). They provide an \((S, s)\) model of durable stock adjustment by heterogeneous consumers with monopoly pricing by firms, also in a partial equilibrium environment. Our approach to modeling demand is very different which allows us to address the cases of oligopoly and monopolistic competition. Esteban and Shum (2007) study price and quantity dynamics in an oligopolistic environment with secondary markets for the case of automobiles. Their focus is on measuring the competitive importance of the secondary market.

Section 3.2 describes the demand for durable goods. Section 3.3 derives results for pricing and quantity dynamics for the case of a monopolist and Sections 3.4 and 3.5 analyze dynamics for the case of oligopoly and monopolistic competition.

### 3.2 Durable good demand

Consider an infinitesimal agent deriving instantaneous utility \(U(C_t, D_t; \xi_t)\) from consumption of durable good \(D_t\) and nondurable good \(C_t\) in period \(t\). The parameter \(\xi_t\) represents a demand shock. The time is discrete and the one-period discount factor is \(\beta\). Period \(t\) purchases of the durable good are denoted by \(X_t\) and \(\delta\) is the depreciation rate of the durable, so the dynamics of the stock of durable is described
by
\[ D_t = (1 - \delta)D_{t-1} + X_t. \]  

(3.1)

Denote by \( P_t \) the price of the durable good and by \( P_{Ct} \) the price of the nondurable good, which is being consumed in positive quantities each period. The agent faces one-period gross interest rate equal to \( 1/\beta \). The agent maximizes
\[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, D_t; \xi_t) \]

subject to
\[ \sum_{t=0}^{\infty} \beta^t (P_{Ct}C_t + P_t (D_t - (1 - \delta)D_{t-1})) \leq NPV_0 \]

where \( NPV_0 \) represents the discounted net present value of the agent’s resources\(^1\). The first order conditions for the choice of \( C_t \) and \( D_t \) are
\[ U_1(C_t, D_t; \xi_t) = \lambda_t P_{Ct} \]

and
\[ U_2(C_t, D_t; \xi_t) - \lambda_t P_t + \beta(1 - \delta)E_t\lambda_{t+1}P_{t+1} = 0 \]

where \( \lambda_t \) is the Lagrange multiplier on the period \( t \) budget constraint. In deriving the first order condition we have implicitly assumed that there are no irreversibility of purchases constraints. In other words, the consumer can always sell the remaining stock of the durable good at the market price. Alternatively, we may assume that shocks are small and prices are never so high that the irreversibility constraint \( X_{t+1} \geq 0 \) binds. That is, the representative consumer will want to purchase positive amounts

\(^1\) Generalization to stochastic consumer’s endowment is straightforward.
of the durable good in every period. This will be the case, for example, if \( \delta \) is high enough.

Let us now assume that the period utility function is quasi-linear, \( U(C_t, D_t; \xi_t) \equiv C_t + u(D_t; \xi_t) \), and normalize \( P_C = 1 \). The first order conditions are then equivalent to \( \lambda_t = 1 \), and \(^2\)

\[
u'(D_t; \xi_t) = P_t - \beta(1 - \delta)E_tP_{t+1}
\]  

(3.2)

The marginal utility from an additional unit of the durable good should equal the price of the durable net of the expected future resale value of the undepreciated durable goods stock. In the following sections we use this demand equation as a starting point, specializing at times to linear or constant elasticity demand. Linear demand corresponds to a quadratic utility function \( u \). With a convenient choice of the demand shock \( \xi_t \), the marginal utility may be written as

\[
u'(D_t; \xi_t) = a + \xi_t - bD_t.
\]

for some parameters \( a \) and \( b > 0 \). Constant elasticity demand\(^3\) corresponds to \( u \) being a concave power function. The demand shock \( \xi_t \) is chosen so that

\[
u'(D_t; \xi_t) = \xi_tD_t^{-1/\sigma},
\]

where \( \sigma \) is the demand elasticity in case of no durability, i.e., \( \delta = 1 \).

In the next section we introduce the firm that produces the durable good and its

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\(^2\) In general, for functions of multiple variables, prime is used to denote the partial derivative with respect to the first argument.

\(^3\) This choice of terminology requires caution. The demand function does not have constant elasticity with respect to the price \( P_t \), but with respect to the price at which the durable goods could be, in principle, rented, namely \( P_t - \beta(1 - \delta)E_tP_{t+1} \).
price setting decision. We assume that the firm’s production function is linear, with constant marginal costs $W_t$ that can vary over time.

### 3.3 Durable good monopoly

#### 3.3.1 The commitment case as a benchmark

Consider a monopolistic firm producing durable goods at a marginal cost $W_t$ in period $t$, which can commit to a sequence of prices $\{P_t\}_t^\infty$ of its choice. The prices may depend on the state of the world. The sequence of prices is chosen in period $t = 0$ to maximize the discounted net present value of profits

$$
E_0 \sum_{t=0}^\infty \beta^t (P_t - W_t) X_t,
$$

where $X_t$ satisfies (3.1) and $D_t$ satisfies (3.2). For convenience, we denote by $(1 - \delta)D_{-1}$ the stock of the durable good the consumers had at the beginning of period $t = 0$ before making any purchases in that period. The following lemma characterizes the optimal choice of prices by the monopolist.

**Lemma 1 (Monopoly, pricing with commitment):** Provided that the initial condition is $D_{-1} = 0$, the durable good monopolist’s optimal pricing with commitment satisfies

$$
P_t = E_t \sum_{j=0}^\infty \beta^j (1 - \delta)^j \frac{\sigma_{t+j}}{\sigma_{t+j} - 1} \left( W_{t+j} - \beta(1 - \delta)E_{t+j}W_{t+j+1} \right),
$$

where

$$
\sigma_t \equiv \sigma(D_t; \xi_t) = -\frac{u'(D_t; \xi_t)}{u''(D_t; \xi_t)D_t'},
$$

where
and the level of demand $D_t$ is determined by

$$u'(D_t; \xi_t) = \frac{\sigma_t}{\sigma_t - 1} (W_t - \beta(1 - \delta)E_tW_{t+1})$$

**Proof:** The Lagrangian for the firm’s problem may be written as

$$\mathcal{L}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ (P_t - W_t)(D_t - (1 - \delta)D_{t-1}) + \lambda_t \left( u'(D_t; \xi_t) - P_t + \beta(1 - \delta)P_{t+1} \right) \right],$$

where $\lambda_t$ is the Lagrange multiplier for the demand condition (3.2). The optimality conditions are

$$(D_t - \lambda_t) - (1 - \delta)D_{t-1} = 0 \text{ for } t = 0,$$

$$(D_t - \lambda_t) - (1 - \delta)(D_{t-1} - \lambda_{t-1}) = 0 \text{ for } t > 0,$$

$$(P_t - W_t) - \beta(1 - \delta)E_t(D_{t+1} - W_{t+1}) + \lambda_t u''(D_t) = 0 \text{ for all } t.$$  

When $D_{t-1} = 0$, the optimal path of $D_t$ is $D_t = \lambda_t$, and the optimal path of $P_t$ satisfies

$$(P_t - W_t) - \beta(1 - \delta)E_t(P_{t+1} - W_{t+1}) = -D_t u''(D_t) \quad (3.3)$$

Using the demand equation (3.2) and rearranging terms yields the demand condition in the proposition:

$$u'(D_t; \xi_t) = \frac{\sigma_t}{\sigma_t - 1} (W_t - \beta(1 - \delta)E_tW_{t+1})$$

Recursively substituting for $P_{t+j}$ in (3.2), letting $E_t \beta^j(1 - \delta)/P_{t+j} \rightarrow 0$ as $j \rightarrow \infty$, and substituting for $u''(D_t; \xi_t)$ using the previous equation one arrives at the price equation in the proposition. ■
Note that in the case of competitive pricing, \( P_t = W_t \), and \( D_t \) evolves according to \( u'(D_t; \xi_t) = W_t - \beta(1 - \delta)W_{t+1} \), which differs from the monopoly pricing solution by the factor \( \sigma_t / (\sigma_t - 1) \).

In general, it follows from the proposition that there is no endogenous dynamics in prices. Prices set at time \( t \) do not depend on lagged prices, only on current and future costs. If the costs \( W_t \) follow an AR(1) process with mean \( \bar{W} \) and persistence \( \phi_W \), the price sequence satisfies

\[
P_t = (1 - \beta(1 - \delta)) \sum_{j=0}^{\infty} \beta^j(1 - \delta)^j \mathbb{E}_t \left\{ \frac{\sigma_{t+j}}{\sigma_{t+j} - 1} \left[ \bar{W} + (W_{t+j} - \bar{W}) \frac{1 - \beta(1 - \delta)\phi_W}{1 - \beta(1 - \delta)} \right] \right\}.
\]

For constant elasticity demand, \( \sigma_t \equiv \sigma \), and arbitrary processes for cost and demand shocks, the optimal pricing policy is the standard constant markup pricing:

\[
P_t = \frac{\sigma}{\sigma - 1} W_t.
\]

This pricing has the following properties: (a) complete instantaneous pass-through; (b) independence from demand shocks; and (c) independence from the depreciation rate \( \delta \) and length of time period (inversely related to \( \beta \)). It coincides with the optimal nondurable pricing policy. This will be our benchmark for comparison in what follows.

With linear demand \( u'(D_t; \xi_t) = a + \zeta - bD_t \) and AR(1) processes for demand and cost shocks (with persistence parameters \( \phi_{\xi} \) and \( \phi_W \)), we have

\[
a + \zeta_t - 2bd_t = \bar{W} + (W_t - \bar{W})[1 - \phi_W \beta(1 - \delta)],
\]

\[
\sigma_t = \frac{a + \zeta_t}{bD_t} - 1,
\]

\[
P_t = \frac{1}{2} \left[ \frac{a}{1 - \beta(1 - \delta)} + (W_t - \bar{W}) + \frac{\zeta_t}{1 - \phi_W \beta(1 - \delta)} \right].
\]
Here the mean of $\xi_t$ is set to zero.

### 3.3.2 Discretionary pricing of durable goods

It is well known that the commitment solution for the monopolist is time inconsistent. The demand for the durable good depends on its expected future price. In the current period the monopolist would benefit if the consumers believed that the future prices of the durable good would be high, but in the next period, the monopolist would like to lower the price in order to increase sales. In the absence of the monopolist’s ability to commit to high future prices, the consumers will base their current purchases on their expectation of low future prices. This impedes the producer’s ability to capture the full potential monopoly rent.

In this subsection we evaluate the case of such discretionary pricing. In each period the monopolist sets prices taking as given the residual demand for the good. The monopolistic firm does not internalize the effect its expected pricing in the current period had on demand in the previous period. We solve for Markov perfect equilibria.

In the case under consideration, the demand for the product in each period is still given by (3.2), where $\mathbb{E}_tP_{t+1}$ are the expectations of consumers about the pricing policy of the monopolist. There are two exogenous state variables $W_t$ and $\xi_t$, both of which follow Markov processes. We restrict attention to Markov perfect equilibria where the only endogenous state variable is $D_{t-1}$. In other words, we assume that only the current level of the ‘physical’ state variable matters for decisions economic agents, not the full history leading to this level. A change in $D_t$ affects the policy of the firm in future periods and consumers take this into account when they demand the durable good today.
Formally, denote by $V(D_{t-1}; W_t, \xi_t)$ the value of the firm as a function of the endogenous and exogenous state variables. The value satisfies the Bellman equation

$$V(D_{t-1}; W_t, \xi_t) = \max_{P_t, D_t} \left\{ (P_t - W_t) \left( D_t - (1 - \delta)D_{t-1} \right) + \beta \mathbb{E}_t V(D_t; W_{t+1}, \xi_{t+1}) \right\} \quad (3.4)$$

subject to the demand condition

$$u'(D_t; \xi_t) = P_t - \beta(1 - \delta)\mathbb{E}_t P_{t+1}, \quad (3.5)$$

We represent the optimal behavior of the firm and the consumers using policy functions $d, p,$ and $f$ as follows:

$$P_t = p(D_{t-1}; W_t, \xi_t)$$

$$D_t = d(P_t; W_t, \xi_t) = d(p(D_{t-1}; W_t, \xi_t); W_t, \xi_t) \equiv f(D_{t-1}; W_t, \xi_t)$$

The first order condition for the choice of $P$ gives

$$D_t - (1 - \delta)D_{t-1} + (P_t - W_t) d'(P_t; W_t, \xi_t) + \beta d'(P_t; W_t, \xi_t) \mathbb{E}_t V'(d(P_t; W_t, \xi_t); W_{t+1}, \xi_{t+1}) = 0,$$

which combined with the envelope condition

$$V'(D_{t-1}; W_t, \xi_t) = -(1 - \delta)(P_t - W_t)$$

yields

$$(D_t - (1 - \delta)D_{t-1}) + [(P_t - W_t) - \beta(1 - \delta)\mathbb{E}_t (P_{t+1} - W_{t+1})] d'(P_t; W_t, \xi_t) = 0. \quad (3.6)$$
Substituting \(d(P_t; W_t, \xi_t)\) for \(D_t\) and \(p(d(P_t; W_t, \xi_t); W_{t+1}, \xi_{t+1})\) for \(P_{t+1}\) in (3.5) and differentiating with respect to \(P_t\) leads to

\[
d'(P_t; \xi_t) = \frac{1}{u''(D_t; \xi_t) + \beta(1 - \delta)E_t p'(D_t; W_{t+1}, \xi_{t+1})}
\]

(3.7)

The following lemma characterizes the equilibrium pricing policy without commitment.

Lemma 2 (Monopoly, pricing without commitment): (i) The dynamics of prices and quantities of the durable good are described by the following dynamic system:

\[
(P_t - W_t) - \beta(1 - \delta)E_t (P_{t+1} - W_{t+1}) = (D_t - (1 - \delta)D_{t-1}) (u''(D_t; \xi_t) + \beta(1 - \delta)E_t p'(D_t; W_{t+1}, \xi_{t+1}))
\]

\[
u'(D_t; \xi_t) = p(D_{t-1}; W_t, \xi_t) - \beta(1 - \delta)E_t p(D_t; W_{t+1}, \xi_{t+1}).
\]

This system simultaneously determines the equilibrium dynamics of \(D_t\) and the optimal policy function \(p\).

(ii) The policy functions satisfy \(p'(D_{t-1}; W_t, \xi_t) < 0\) and \(f'(D_{t-1}; W_t, \xi_t) > 0\).

Proof: Equations (3.8) and (3.9) follow from (3.6), (3.7), and (3.5). The conditions \(p'(D_{t-1}; W_t, \xi_t) < 0\) and \(f'(D_{t-1}; W_t, \xi_t) > 0\) are derived in the appendix.

Condition (3.8), or (3.6), has an intuitive interpretation: it is the firm’s optimality condition that can be obtained using a perturbation argument. We can rewrite it as

\[
P_t - \beta(1 - \delta)E_t P_{t+1} + \frac{1}{d'(P_t; W_t, \xi_t)} X_t = W_t - \beta(1 - \delta)E_t W_{t+1}.
\]

The left hand side is the marginal revenue associated with increasing the quantity
\[ X_t = D_t - (1 - \delta)D_{t-1} \] sold in period \( t \) and reducing the quantity \( X_{t+1} \) sold in the next period in a way that leaves \( D_{t+1} \) unchanged. The first two terms represent the direct gain from markups, which would be present even in competitive markets. The third term captures the loss in the monopolist’s profit margin due to the negative movement in price \( P_t \). The right hand side, of course, represents the marginal cost corresponding to this small change in production.

Lemma 2 fully characterizes the equilibrium dynamics. The equilibrium pricing is suboptimal in the sense that it no longer satisfies the conditions of Lemma 1. To see this observe the two extra expressions in (3.8) compared to (3.3): \( D_{t-1} \) and \( p'(D_t; W_{t+1}, \xi_{t+1}) \). Both are in general non-zero and they are directly related to the time inconsistency of the monopolistic pricing. In particular, \( D_{t-1} \) appears because the monopolist does not internalize the effect of the pricing policy on demand last period. The expression \( p'(D_t; W_{t+1}, \xi_{t+1}) \) represents the fact that the monopolist can partially affect its future pricing policy through the state variable.

**Result 3** In the absence of shocks to cost and demand \( (W_t = \bar{W}, \xi_t = \bar{\xi}) \) the steady state price is given by

\[
\bar{P} = \frac{\sigma}{\sigma - \delta \bar{\omega}} \bar{W}
\]

where \( \sigma \equiv \sigma(\bar{D}; \bar{\xi}) \), steady state consumption \( \bar{D} \) satisfies \( u'(\bar{D}) = (1 - \beta(1 - \delta)) \bar{P} \), and \( \bar{\omega} \equiv 1 + \beta (1 - \delta)p'(\bar{D}; \bar{W}, \bar{\xi}) / u''(\bar{D}; \bar{\xi}) \). As a corollary, there is marginal cost pricing in the long run if the good is perfectly durable, i.e., \( \bar{P} = \bar{W} \) when \( \delta = 0 \).

**Proof:** This result directly follows from (3.8)-(3.9).

Note that for \( \delta > 0 \) the value of steady state markup is related to the out-of-steady-state behavior of prices.
Solution in the case of quadratic utility

Bond and Samuelson (1984) studied the durable good monopolist problem in the case of quadratic utility. They demonstrated the following properties of the steady state: (i) in the case of zero depreciation rate, marginal cost pricing is reached in the long run; (ii) for a non-zero depreciation rate, price is above marginal cost in the long-run if the time period has non-zero length; (iii) in the limit of zero length of the time period, the durable good price is equal to the marginal cost, independently of δ, i.e., the Coase conjecture holds. In the present work, we additionally study the dynamics of pricing.

Lemma 4 (Linear pricing under quadratic utility): Let \( W_t \) and \( \xi_t \) follow AR(1) processes with means \( \bar{W} \) and zero, respectively. With quadratic utility, \( u'(D_t; \xi_t) = a - bD_t + \xi_t \), there exists a linear solution to the pricing equations (3.8)-(3.9):

\[
p(D_{t-1}; W_t, \xi_t) = \bar{P} - \alpha(D_{t-1} - \bar{D}) + \gamma(W_t - \bar{W}) + \lambda \xi_t,
\]

(3.10)

In addition, the dynamics of the state variable is also linear and satisfies

\[
D_t = \bar{D} + \phi(D_{t-1} - \bar{D}) - \psi(W_t - \bar{W}) + \eta \xi_t.
\]

(3.11)

Here \( \alpha, \gamma, \lambda, \phi, \psi, \) and \( \eta \) are constants.

Proof: In this case \( u''(D_t; \xi_t) = -b \). Substituting the conjectured linear policy rules (3.10) and (3.11) into the pricing equations (3.8) and (3.9) and comparing coefficients of different terms eight conditions for the eight unknown parameters. The solution
to this system of equations is

\[ \begin{align*}
\alpha &= \frac{b(\zeta-1)}{\beta(1-\delta)} > 0, \\
\gamma &= \frac{\bar{\zeta}}{1+\bar{\zeta}} \in (1/2, 1), \\
\lambda &= \frac{1}{1+\bar{\zeta}} \frac{1-\beta(1-\delta)\phi_\xi}{1-\beta(1-\delta)} > 0, \\
\bar{p} &= \frac{\bar{W}}{1-\beta(1-\delta)+\delta \xi} + \frac{a}{1-\beta(1-\delta) - \beta(1-\delta)+\delta \xi} - \beta(1-\delta)\phi_W(W_{t-1} - \bar{W}) \\
\bar{D} &= \frac{1}{b} \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)+\delta \xi}.
\end{align*} \]

where \( \bar{\zeta} \equiv (1 - \beta(1 - \delta)^2)^{-1/2} > 1 \), and \( \phi_W \) and \( \phi_\xi \) are the persistence parameters of the process \( W_t \) and \( \bar{\zeta}_t \), respectively. It is straightforward to verify that with these parameters, the dynamic equations are satisfied.

Note that equations (3.10) and (3.11) imply the following process for prices:

\[ \begin{align*}
P_t - \bar{P} &= \phi (P_{t-1} - \bar{P}) + \gamma (W_t - \bar{W}) + \lambda \bar{\zeta}_t - \phi \beta(1-\delta)\gamma \phi_W(W_{t-1} - \bar{W}) \\
&\quad - \phi(1 + \lambda \beta(1-\delta)\phi_\xi)\bar{\zeta}_{t-1}.
\end{align*} \]

**Result 5** In this environment prices adjust slowly over time in response to shocks (even one-time permanent shocks) and pass-through is incomplete. As the durability of the good declines \( (\delta \to 1) \), the persistence parameter \( \phi \) approaches zero, i.e., in the nondurable limit there is no endogenous dynamics.

1. \( D_t \) increases over time, while \( P_t \) and markup decrease over time.

2. Markup and price increase in response to a positive demand shock (procyclical markup).

3. Markup decreases and price increases in response to a positive cost shock (countercyclical markup).

The fact that markups decrease in response to a positive cost shock has implications for the literature on incomplete exchange rate pass-through. Most traded goods
are durable in nature. The fact that pass-through is incomplete in the long-run is frequently attributed to strategic complementarities in pricing that prevents a firm from raising its price in response to cost shocks as this causes the elasticity of demand it faces to rise. Adding the assumption of frictions in price adjustment then generates dynamics in pass-through.

We have seen that in the case of durable goods with discretion in pricing one obtains pass-through dynamics even in the flexible price case, and incomplete pass-through even in the case the absence of standard strategic complementarities in pricing. In a sense, in the case under consideration, there are strategic complementarities over time that arise from the firm competing with itself.

The solution here also contrasts with the marginal cost pricing case of durable goods. It exhibits smaller price volatility and quantity volatility.

General utility functions

Here we present a compact equation that determines the transition function $f$ in the absence of demand shocks and cost shocks. Note that for small deviations from the steady state $\bar{D}$ the persistence parameter is given by $f'(\bar{D})$.

Lemma 6 (Transition function for general utility): The transition function $f$, for any $D$, satisfies

$$
(1 - \beta (1 - \delta)) \frac{W - u'(f(D))}{f(D) - (1 - \delta)D} - u''(f(D))
\]

$$

Moreover, with the knowledge of $f$, the consumer reaction function $d$ may be recov-
\[ f(d(P)) = \frac{P - u'(d(P))}{\beta(1 - \delta)} \]  

(3.13)

**Proof:** Notice that (3.13) is an immediate consequence of the demand equation \( P - u'(d(P)) = \beta(1 - \delta) p(d(P)) \). We just need to demonstrate the result (3.12). We start with (3.6) adapted to the case under consideration, with time indices suppressed:

\[
\left(d(P) - (1 - \delta)f^{-1}(d(P))\right) + [(P - W) - \beta(1 - \delta)(p(d(P)) - W)]d'(P) = 0.
\]

The demand equation \( P - \beta(1 - \delta)p(d(P)) = u'(d(P)) \) then implies

\[
d(P) - (1 - \delta)f^{-1}(d(P)) = [(1 - \beta(1 - \delta))W - u'(d(P))]d'(P).
\]

Applying the inverse function differentiation theorem \( d^{-1'}(D) = 1/d'(d^{-1}(D)) \), we get

\[
d^{-1'}(D) = \frac{(1 - \beta(1 - \delta))W - u'(D)}{D - (1 - \delta)f^{-1}(D)}.
\]

Integrating both sides of this equation gives

\[
d^{-1}(D) - \bar{p} = \int_{D}^{D} \frac{(1 - \beta(1 - \delta))W - u'(D)}{D - (1 - \delta)f^{-1}(D)}dD.
\]

(3.14)

This equation must hold for any \( D \). Using this last equation at a different point, and noting that \( d^{-1}(D_+) = p(D) \), we get

\[
p(D) - \bar{p} = \int_{D}^{D_+} \frac{(1 - \beta(1 - \delta))W - u'(D)}{D - (1 - \delta)f^{-1}(D)}d\tilde{D}.
\]

(3.15)

Multiplying (3.15) by \(-\beta(1 - \delta)\) and adding the resulting equation to (3.14) yields
\[
\begin{align*}
&= u'(D) - (1 - \beta (1 - \delta)) \bar{P} \\
&= \int_{D}^{D} \frac{(1 - \beta (1 - \delta)) W - u'(\bar{D})}{D - (1 - \delta) f^{-1}(D)} d\bar{D} - \beta (1 - \delta) \int_{D}^{f(D)} \frac{(1 - \beta (1 - \delta)) W - u'(\bar{D})}{D - (1 - \delta) f^{-1}(D)} d\bar{D}.
\end{align*}
\]

Here we simplified the left hand using the demand equation \( d^{-1}(D) - \beta (1 - \delta) p(D) = u'(D) \). Differentiation with respect to \( D \) (and shifting the time period under consideration) leads to the final result (3.12).

**Numerical solution**

We further study the properties of the dynamics of durable good pricing without commitment in the special case of constant elasticity utility by means of a numerical solution. The solution to the dynamic system (3.8)–(3.9) does not have a simple characterization in this case, and we obtain the model’s solution using value function iteration.

Specifically, we start with a guess for the value function, \( V(\cdot) \), and pricing rule, \( p(\cdot) \). Given the pricing rule, we can use demand (3.2) express current period price as a function of current period stock \( D \):

\[
P = u'(D) + \beta (1 - \delta) \mathbb{E} p(D),
\]

where we have suppressed demand and cost shocks from notation (including in the expectation). Substituting this expression into the value function, we can write the
value of the firm recursively:

\[
\tilde{V}(D_{-}) = \max_{D} \left\{ (u'(D) + \beta(1 - \delta)E p(D) - W)(D - (1 - \delta)D_{-}) + \beta E V(D) \right\},
\]

(3.16)

where \(D_{-}\) is the state variable—the stock of the durable good last period. The arg max of this problem is the state variable transition function, \(D = \tilde{f}(D_{-})\). From this we can update the pricing rule according to:

\[
\tilde{p}(D_{-}) = u'(\tilde{f}(D_{-})) + \beta(1 - \delta)E p(\tilde{f}(D_{-})).
\]

Hence, on each iteration, given the initial guesses \(V(\cdot)\) and \(p(\cdot)\), we obtain a new value function \(\tilde{V}(\cdot)\) and a new pricing rule \(\tilde{p}(\cdot)\). We repeat this procedure until convergence. In order to obtain convergence, we apply polynomial smoothing to \(\tilde{V}(\cdot)\) and \(\tilde{p}(\cdot)\) on each iteration.

In this numerical solution, we focus on the case of constant elasticity demand with \(\sigma = 2\). Larger values of \(\sigma\) mute the endogenous dynamics of durable monopolist prices. We choose the remaining two parameters—the discount rate and depreciation rate—at \(\beta = 0.9\) and \(\delta = 0.2\) respectively. This roughly correspond to a 2.5-year period. The reason for the choice of such a long time period is technical complications with numerical convergence for smaller values of \(\delta\), and future versions of this work will address this technical issue.

Simulations confirm qualitative approximation results above for price and quantity dynamics. We start by exploring the dynamic path towards the steady state when both cost and demand shocks are switched off (\(W_t \equiv \tilde{\xi}_t \equiv 1\)), and the initial stock of the durable good is zero (\(D_{-1} = 0\)). Figure 3.1 reports the path of prices in the
right panel and the path of quantities in the left panel. We contrast the equilibrium dynamics without commitment with the case of marginal-cost pricing and monopoly-pricing with commitment. In both of these benchmark cases there is no dynamics in prices and durable stock jumps instantaneously to its steady state level. The dynamics without commitment is substantially different: prices gradually decline as stock gradually rises, reaching steady state only after a number of periods. This illustrates endogenous dynamics in this case and the role of the durable stock as the state variable.

Next, in Figures 3.2–3.3, we consider the response of monopolist price, markup and durable stock to an unanticipated permanent cost and demand shocks. Confirming our theoretical findings, markup is decreasing in response to a cost rise, and increasing in response to a positive demand shock. This endogenous response of markup and price dampens the short-run response of quantities to both shocks. As a result, quantities adjust only gradually to both shocks, which is contrasted with the immediate adjustment of quantities under both marginal-cost and constant-markup
Figure 3.2: Response to a one-time cost shock

Figure 3.3: Response to a one-time demand shock
(the case of commitment) pricing.

Table 3.1: Equilibrium Dynamics under Cost Shocks

<table>
<thead>
<tr>
<th>log(·)</th>
<th>σ (%)</th>
<th>ρ</th>
<th>corr(·, log (W_t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage, (W_t)</td>
<td>4.9</td>
<td>0.80</td>
<td>1.00</td>
</tr>
<tr>
<td>Price, (P_t)</td>
<td>5.1</td>
<td>0.90</td>
<td>0.88</td>
</tr>
<tr>
<td>Markup, (P_t/W_t)</td>
<td>2.2</td>
<td>0.69</td>
<td>-0.19</td>
</tr>
</tbody>
</table>

Durable stock, \(D_t\)
- constant markup: 15.5, 0.79, -0.99
- discretion: 12.2, 0.95, -0.75
- ratio (disc/comm): 0.29

Durable purchases, \(X_t\)
- constant markup: 70.7, -0.08, -0.31
- discretion: 21.4, 0.57, -0.91
- ratio (disc/comm): 0.16

Finally, we simulate the partial equilibrium dynamics under stochastic cost and demand shocks. First, we consider stochastic cost shocks evolving according to a discretized version of an AR(1) with standard deviation of innovation of 5% and a persistence of 0.8. Table 3.1 reports the statistical properties prices and quantities in this dynamic equilibrium. Markup is indeed countercyclical and price exhibits endogenous persistence, in excess of that of the exogenous cost process. Furthermore, durable good purchases are mildly negatively correlated under constant-markup pricing, while they become strongly positively autocorrelated under solution without commitment. This is an empirically appealing property of durable pricing without commitment, which allows to obtain realistic dynamics of durable purchases even in the absence of adjustment costs. Lastly, endogenous markup variation without commitment substantially reduces the volatility of both durable purchases and durable stock.

Table 3.2 reports the pass-through coefficients from the regression of prices on costs, with and without the lagged cost variable. The upper panel runs the pass-
Pass-through of Cost Shocks

<table>
<thead>
<tr>
<th></th>
<th>( \log W_t )</th>
<th>( \log W_{t-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \log P_t )</td>
<td>0.91</td>
<td></td>
</tr>
<tr>
<td>( \log P_t )</td>
<td>0.65</td>
<td>0.34</td>
</tr>
<tr>
<td>( \Delta \log W_t )</td>
<td>0.61</td>
<td>( \Delta \log W_{t-1} )</td>
</tr>
<tr>
<td>( \Delta \log P_t )</td>
<td>0.61</td>
<td>0.15</td>
</tr>
</tbody>
</table>

through regression in levels, while the lower panel produces results in differences. Pass-through is incomplete (91% in levels and 61% in differences) with over 2/3 of pass-through happening on impact and the remaining pass-through after one period.

Table 3.3: Equilibrium Dynamics under Demand Shocks

<table>
<thead>
<tr>
<th>Log(·)</th>
<th>( \sigma ) (%)</th>
<th>( \rho )</th>
<th>corr(·, log(( \xi_t )) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand, ( \xi_t )</td>
<td>4.8</td>
<td>0.77</td>
<td>1.00</td>
</tr>
<tr>
<td>Price and markup, ( P_t / W )</td>
<td>1.9</td>
<td>0.79</td>
<td>0.18</td>
</tr>
<tr>
<td>Durable stock, ( D_t )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>— constant markup</td>
<td>9.7</td>
<td>0.77</td>
<td>1.00</td>
</tr>
<tr>
<td>— discretion</td>
<td>7.2</td>
<td>0.94</td>
<td>0.66</td>
</tr>
<tr>
<td>— ratio (disc/comm)</td>
<td></td>
<td></td>
<td>-0.22</td>
</tr>
<tr>
<td>Durable purchases, ( X_t )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>— constant markup</td>
<td>36.1</td>
<td>-0.03</td>
<td>0.91</td>
</tr>
<tr>
<td>— discretion</td>
<td>13.6</td>
<td>0.56</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Finally, we consider the dynamic equilibrium with demand shocks. We assume that demand shocks follow a discretized \( AR(1) \) process also with standard deviation of innovation of 5% and persistence of 0.8. Table 3.3 reports the statistical properties of the dynamics of prices and quantities in this stochastic equilibrium. The markup is procyclical in this case, with a standard deviation of about 2% and persistence of about 0.8. This again results in positively autocorrelated durable purchases in contrast with \( iid \) durable purchases under marginal-cost or constant-markup pricing. In addition, both durable purchases and durable stock is less volatile under pricing.
without commitment, as in the case of cost shocks.

3.3.3 Extension: Calvo sticky price setting

Result 7 (Equivalence of Calvo and Flexible Pricing): The solution of a durable-good monopolist pricing problem under Calvo price stickiness with probability of price non-adjustment \( \theta \) is identical to the problem of a flexible price durable-good monopolist under the following substitution of primitives: \( \beta \rightarrow \beta_C, \delta \rightarrow \delta_C, u \rightarrow u_C \), where

\[
\begin{align*}
\beta_C & \equiv \beta \frac{1-\theta}{1-\beta\theta}, \\
\delta_C & \equiv 1 - \frac{1-\beta\theta}{1-\beta\theta(1-\delta)} (1-\delta), \\
u_C(D) & \equiv \frac{1}{1-\beta\theta(1-\delta)} u(D).
\end{align*}
\] (3.17)

Proof: We have seen that with flexible prices the dynamics can be summarized as

\[
V(D_\cdot) = \max_P \{(P - W) (d(P) - (1-\delta)D_\cdot) + \beta V(d(P))\},
\] (3.18)

which yields policy function \( p(D_\cdot) \), and where the consumer reaction function \( d(P) \) is implicitly defined by the consumer first order condition

\[
u'(d(P)) = P - \beta(1-\delta)p(d(P)).
\] (3.19)

To keep notation simple, we suppressed the time indices, replacing \((P_t, D_{t-1})\) by \((P, D_\cdot)\).

Now let us consider the case of Calvo price setting. Denote by \( 1 - \theta \) the probability that the firm is allowed to adjust prices in any definite period. The consumer first
order condition will become

\[ u'(d(P)) = P - \beta (1 - \delta) (\theta P + (1 - \theta) p(d(P))), \]

where \( p \) is again the firm’s desired price as the function of the state variable. Using definitions (3.17), this may be rewritten as

\[ u'_C (d(P)) = P - \beta C (1 - \delta_C) p (d(P)). \] (3.20)

Note that demand as a function of \( P \) is the same irrespective of whether the firm adjusted the price this period: consumers take the current price as given and the expected price next period (for a given stock of durables) does not depend on whether the firm adjusted the price in the current period.

The problem of the firm in a period of price adjustment is

\[ V(D_-) = \max_p \{ (P - W)(d(P) - (1 - \delta)D_-) + \beta (1 - \theta) V(d(P)) + \beta \theta U(P) \}, \]

where \( U(P) \) is the value of the firm if the price is not adjusted; i.e., \( U(P) \) is given by

\[ U(P) = (P - W)\delta d(P) + \beta (1 - \theta) V(d(P)) + \beta \theta U(P). \]

Note that if the firm does not adjust prices, consumers will consume the same amount as in the previous period. Combining the two expressions, leads to the Bellman equation

\[ V(D_-) = \max_p \left\{ (P - W) \left[ \frac{1 - \beta (1 - \delta) \theta}{1 - \beta \theta} d(P) - (1 - \delta)D_- \right] + \frac{\beta (1 - \theta)}{1 - \beta \theta} V(d(P)) \right\}. \]
Recalling the definitions (3.17), we see that problem is equivalent to

$$\tilde{V}(D) = \max_P \left\{ \left( P - W \right) (d(P) - (1 - \delta C)D) + \beta_C \tilde{V}(d(P)) \right\},$$

(3.21)

with

$$\tilde{V}(D) = \frac{1 - \beta \theta}{1 - \beta (1 - \delta) \theta} V(D).$$

Equations (3.20) and (3.21) are the same as (3.19) and (3.18) with the replacement $\beta \rightarrow \beta C$, $\delta \rightarrow \delta C$, $u \rightarrow u C, V(D) \rightarrow \tilde{V}(D)$. This implies the equivalence of the
dynamics with flexible prices and with Calvo price setting stated above.

3.4 Durable good oligopoly

We have analyzed the price dynamics of durable goods in the case of monopoly. In this section, we will investigate the case of Cournot competition of $N$ symmetric oligopolistic firms producing a homogenous durable good. The firms are unable to commit to future production policy. The production decision of each firm is influenced by two different forces: competition with the other firms and inter-temporal competition with itself, since the future decision makers at the firm do not have the same objectives. Macroeconomic literature has focused on the first effect, assuming that for firms production decisions are based on the current competition, see (e.g., Barsky, House, and Kimball, 2007). Here we model both forces jointly. As we will see, both of them push prices closer to marginal cost.

The stock of the durable good is the only state variable. For simplicity, we do not consider stochastic cost shocks and demand shocks here, although the analysis can be extended to parallel our discussion of monopoly. In this simpler setting, one can still analyze the endogenous price dynamics in response to permanent shocks and
identify the value of the persistence parameter. The following lemma characterizes the price dynamics in the case under consideration.

Lemma 8 (Durable-good oligopoly equilibrium dynamics): The equilibrium in the durable-good Cournot oligopoly market with $N$ symmetric firms with constant marginal cost $W$ is described by the following system:

$$
(P_t - W) - \beta (P_{t+1} - W) \left( \frac{1 - \delta}{N} + \frac{N - 1}{N} f'(D_t) \right) = -\frac{D_t - (1 - \delta)D_{t-1}}{N} \left( u''(D_t) + \beta (1 - \delta) p'(D_t) \right)
$$

(3.22)

where

$$
u'(D_t) = P_t - \beta (1 - \delta) P_{t+1},
$$

(3.23)

where

$$
D_t = f(D_{t-1}), \quad P_t = p(D_{t-1}).
$$

**Proof:** Consumer optimization immediately implies

$$
P_t = u'(D_t) + \beta (1 - \delta) p(D_t),
$$

(3.24)

where $p(D_t)$ describes the dependence of the price of the durable good in the next period on the stock of the good in the current period. This is the second equation of the lemma. Denote by $x(D_{t+1})$ the equilibrium strategy (i.e., the amount produced) of every oligopolist a function of the state variable. Suppose that in period $t$ one firm deviates from its equilibrium strategy $x(D_{t-1})$, and produces $\hat{x}_t$ instead. In this case

$$
D_t = (1 - \delta) D_{t-1} + \hat{x}_t + (N - 1)x(D_{t-1}).
$$

(3.25)
Since the firm is free to choose its optimal level of production, its value satisfies the Bellman equation

\[ V(D_{t-1}) = \max_{\tilde{X}_t} \left\{ (P_t(\tilde{x}_t) - W) \tilde{X}_t + \beta V(D_t(\tilde{X}_t)) \right\}, \]

where the dependence of \( P_t \) and \( D_t \) on \( \tilde{X}_t \) is given by equations (3.24) and (3.25). Denote by \( \lambda \) and \( \mu \) the Lagrange multipliers on those two equations. The envelope condition and the first order conditions corresponding to the choice of \( \tilde{X}_t, P_t \) and \( D_t \) may be written as

\[
V'(D_{t-1}) = -\mu \left( (1 - \delta) + (N - 1) x'(D_{t-1}) \right),
\]

\[
P_t - W = \mu,
\]

\[
\beta V'(D_t) + \mu = -\lambda \left( u''(D_t) + \beta(1 - \delta)p'(D_t) \right),
\]

\[
\tilde{X}_t = \lambda.
\]

Substituting out the Lagrange multipliers and imposing the equilibrium requirement \( \tilde{X}_t = x(D_{t-1}) \) yields

\[
(P_t - W) - \beta (P_{t+1} - W) \left( (1 - \delta) + (N - 1) x'(D_t) \right) = -x(D_{t-1}) \left( u''(D_t) + \beta(1 - \delta)p'(D_t) \right)
\]

Just like (3.8), this condition can be justified by a simple perturbation argument (see the discussion following Lemma 2). The firm sells a little bit more in period \( t \) and a little less in period \( t + 1 \), with this amounts chosen in a way that lead to the original equilibrium path from period \( t + 2 \) onward. In choosing the quantity deviations for the perturbation argument, one needs to take into account the reaction of the
competitors to the deviation in period \( t \), reflected by the presence of \((N - 1)x'(D_t)\) on the left hand side.

This condition together the following definition of the function \( f \)

\[
D_t = f(D_{t-1}) = (1 - \delta)D_{t-1} + Nx(D_{t-1}), \quad \text{with} \quad f'(D_{t-1}) = (1 - \delta) + Nx'(D_{t-1}).
\]

implies the first equation of the lemma.

For \( N = 1 \), the system of equations in the lemma agrees with the monopoly results. For \( N = 1 \) we observe the following two changes to the system of equations. First, on the right hand side instead of \( X_t = D_t - (1 - \delta)D_{t-1} \) we have \( X_t/N \), which implies that the profit losses of an oligopolist from lower price are shared equally with the \((N - 1)\) competitors. This force is present also in the case of nondurable goods \((\delta = 1)\), and it reduces the price charged by the monopolist. Second, in the spirit of the perturbation argument, if a firm sells one additional unit of the durable good in period \( t \), and wants to compensate for this in period \( t + 1 \) to return to the original equilibrium path, its sales should not drop by \((1 - \delta)\), but by \((1 - \delta)/N\) plus the change in sales by its competitors, reflected by \( x'(D_t)(N - 1)/N \). Since \( x'(D_t) < 0 \), this is less than \((1 - \delta)\).

Lemma 9 (Oligopoly with quadratic utility): With \( N \) symmetric firms, constant marginal cost \( W \), and quadratic utility \( u'(D_t) = a - bD_t \), there exists a ‘linear’ equilibrium of the form:

\[
D_t = \bar{D} + \phi (D_{t-1} - \bar{D}) \tag{3.26}
\]

\[
P_t = \bar{P} - \alpha(D_{t-1} - \bar{D}) \tag{3.27}
\]
which solves the equilibrium system of equations exactly. The positive persistence parameter \( \phi \) satisfies the cubic\(^4\) equation

\[
\beta (N - 1) \phi^3 + \beta (1 - \delta) \phi^2 - (N + 1) \phi + 1 - \delta = 0. \tag{3.28}
\]

With the knowledge of \( \phi \), the parameters \( a, \tilde{D}, \) and \( \tilde{P} \) may be recovered as

\[
\begin{align*}
a & = \frac{b \phi}{\gamma_2} \\
\tilde{D} & = \frac{1}{p} \left( \frac{a}{\gamma_1} - W \right) \frac{\gamma_1 + (N - 1) \gamma_2}{\gamma_1 + (N - 1) \gamma_2 + \frac{p}{\gamma_2}} \\
\tilde{P} & = \frac{a}{\gamma_1} - \frac{1}{\gamma_1} \left( \frac{a}{\gamma_1} - W \right) \frac{\gamma_1 + (N - 1) \gamma_2}{\gamma_1 + (N - 1) \gamma_2 + \frac{p}{\gamma_2}}
\end{align*}
\]

where

\[
\gamma_1 \equiv 1 - \beta (1 - \delta), \quad \gamma_2 \equiv 1 - \beta (1 - \delta) \phi
\]

**Proof:** Plugging the assumed form of the solution (3.26) and (3.27) into the dynamic equations (3.22) and (3.23) and comparing coefficients of different terms leads to the four conditions

\[
\begin{align*}
\frac{a}{\phi} \frac{\delta D}{\phi - W} & = 1 - \beta (1 - \delta) + (N - 1) (1 - \beta \phi) \\
0 & = 1 - \frac{1 - \delta}{\phi} + N - \beta \phi (1 - \delta + (N - 1) \phi) \\
u' (\bar{D}) & = (1 - \beta (1 - \delta)) \bar{P} \\
-u'' (\bar{D}) \frac{\phi}{\phi} & = 1 - \beta (1 - \delta) \phi
\end{align*}
\]

Here we used \( p' (D_{t-1}) = -\alpha \), as implied by (3.27). The second condition may be rewritten as (3.28). The remaining three conditions can be manipulated to give the explicit expressions for \( a, \tilde{D}, \) and \( \tilde{P} \) in the lemma. It is straightforward to check that with these values of the parameters, the dynamic equations (3.22) and (3.23) are

\[\text{4 Of course, any cubic equation can be solved algebraically. For brevity, we will omit the explicit expressions for the solution here.}\]
identically satisfied. ■

**Result 10** Both $\phi$ and $\alpha$ decrease in $N$.

**Proof:** Treat the parameter $N$ in (3.28) as a continuous variable. Implicit function theorem then implies

$$- \frac{d\phi}{dN} = \frac{\phi - \beta\phi^3}{N + 1 - 2\beta(1 - \delta)\phi - 3\beta(N - 1)\phi^2}$$

Since $\phi < 1$, the numerator is clearly positive. Algebraic manipulation may be used to show that the denominator is also positive. This leads to the conclusion that $\phi$ is a decreasing function of $N$. The explicit expression for $\alpha$ is $\alpha = b\phi / (1 - \beta(1 - \delta)\phi)$. This is obviously an increasing function of $\phi$, which implies that $\alpha$ decreases in response to increased $N$. ■

### 3.5 Durable good monopolistic competition

We now consider the case of monopolistic competition with infinitesimal firms producing imperfectly substitutable varieties. The firms cannot commit to a path of future prices. We specialize to the cases where the influence of other firms on the demand for a particular variety can be summarized by a simple sufficient statistic.

Consider the case of a continuum of varieties of durable goods, distinguished by index $i \in \Omega \equiv [0, 1]$. The varieties depreciate at the same rate $\delta$, and the stock of variety $i$ evolves according to

$$D_{it} = X_{it} + (1 - \delta)D_{i,t-1},$$

where $X_{it}$ is quantity of variety $i$ sold in $t$. Denote the price of variety $i$ by $P_{it}$. The
consumer maximizes
\[ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U \left( C_t, \{D_{it}\}_{i \in \Omega} \right) \]
subject to
\[ \sum_{t=0}^{\infty} \beta^t \left( P_{C_t} C_t + \int_{i \in \Omega} P_{it} (D_{it} - (1 - \delta) D_{i,t-1}) \, di \right) = NPV_0 \]
with \( D_{i,-1} \) and \( NPV_0 \) given\(^5\). The consumer’s first order condition is
\[ \frac{\delta U \left( C_t, \{D_{it}\}_{i \in \Omega} \right)}{\delta D_{it}} = \lambda_t P_{it} - \beta (1 - \delta) \mathbb{E}_t \lambda_{t+1} P_{i,t+1}, \]
where \( \delta U \left( C_t, \{D_{it}\}_{i \in \Omega} \right) / \delta D_{it} \) is the functional derivative of \( U \left( C_t, \{D_{it}\}_{i \in \Omega} \right) \) with respect to \( D_{it} \), and \( \lambda_t \equiv U_{1,t} / P_{C_t} \). Consider the case of quasi-linear utility function \( U \left( C_t, \{D_{it}\}_{i \in \Omega} \right) = C_t + u \left( \{D_{it}\}_{i \in \Omega} \right) \). The price of the nondurable good \( P_{C_t} \) will be normalized to one, implying \( \lambda_t = 1 \). The first order condition becomes
\[ \frac{\delta u \left( \{D_{it}\}_{i \in \Omega} \right)}{\delta D_{it}} = P_{it} - \beta (1 - \delta) \mathbb{E}_t P_{i,t+1}. \]
Let us specialize to cases\(^6\) where the influence of competitors on firm \( i \) may be summarized by a sufficient statistic \( D_t \equiv \Psi \left( \{D_{it}\}_{i \in \Omega} \right) \) in the sense that
\[ \frac{\delta u \left( \{D_{it}\}_{i \in \Omega} \right)}{\delta D_{it}} = v(D_{it}; D_t). \]
The demand equation now takes the form
\[ v(D_{it}; D_t) = P_{it} - \beta (1 - \delta) \mathbb{E}_t P_{i,t+1}. \]

---

\(^5\) Generalization to consumer’s stochastic endowment is straightforward.

\(^6\) These include CES utility functions, as well as quadratic utility functions discussed below.
Result 11 For a given process $D_t$, the problem of durable good monopolistic competitor described above is equivalent to the problem of a durable good monopolist with stochastic demand shocks.

For simplicity, let us consider the case without uncertainty: $W_t = W$. The following lemma characterizes the price dynamics.

Lemma 12 (Monopolistic competition, pricing without commitment): The equilibrium prices and quantities satisfy

$$(P_t - W) - \beta(1 - \delta) (P_{t+1} - W) = (D_t - (1 - \delta)D_{t-1}) (v'(D_t; D_t) + \beta(1 - \delta)p'(D_t; D_t)),\nonumber$$

$$v(D_t; D_{t+1}) = P_t - \beta(1 - \delta)P_{t+1}$$

where $D_t \equiv \Psi(\{D_{it}\}_{i \in \Omega})$, and the policy function $p(D_{it}; D_t)$ describes the dependence of $P_{t+1}$ on $D_{it}$ and $D_t$.

Proof: Equation (3.30) has been derived above. Equation (3.29) is an immediate consequence of Lemma 2, with the appropriate renaming of variables. This is because of the equivalence of the problem of durable good monopolistic competitor and durable good monopolist facing shifts in demand. □

Consider now the case of quadratic utility. Our goal is to identify the persistence parameters corresponding to deviations from the steady state. The utility will be parameterized as

$$u(\{D_{it}\}_{i \in \Omega}) = a \int_{i \in \Omega} D_{it}^2 di - \frac{b_1}{2} \int_{i \in \Omega} D_{it}^2 di - \frac{b_2}{2} \left( \int_{i \in \Omega} D_{it} di \right)^2$$
This type of utility function has been used, for example, in Ottaviano, Tabuchi, and Thissse (2002) and Melitz and Ottaviano (2008). The function \( v(D_i; D_t) \) now takes the form

\[
v(D_i; D_t) = a - b_1 D_i - b_2 D_t
\]

with

\[
D_t \equiv \int_{i \in \Omega} D_{it} di
\]

being the average stock of durable goods at time \( t \). The following lemma characterizes the equilibrium path

**Lemma 13 (Monopolistic competition, quadratic utility):** The evolution of prices and quantities is described by

\[
D_{it} = \bar{D} + \phi_k(D_{i,t-1} - \bar{D}) + \phi_a(D_{i,t-1} - D_{t-1}),
\]

\[
P_{it} = \bar{P} - \kappa(D_{i,t-1} - \bar{D}) - \alpha(D_{i,t-1} - D_{t-1}),
\]

\[
D_t \equiv \int_{i \in \Omega} D_{it} di, \quad P_t \equiv \int_{i \in \Omega} P_{it} di.
\]
The parameters are given by

\[ f_a = \frac{1}{\beta(1-\delta)}, \]
\[ a = \frac{b_1}{\beta(1-\delta)} \left( \frac{1}{\sqrt{1-\beta(1-\delta)^2}} - 1 \right), \]
\[ \phi_x = \frac{1-\delta}{1+\frac{b_1+b_2}{b_1}\sqrt{1-\beta(1-\delta)^2}}, \]
\[ \kappa = \frac{(1-\delta)(b_1+b_2)}{1-\beta(1-\delta)^2+\frac{b_1+b_2}{b_1}\sqrt{1-\beta(1-\delta)^2}}, \]
\[ D_s = \frac{a-(1-\beta(1-\delta))W}{\sqrt{1-\beta(1-\delta)^2}+b_1+b_2}, \]
\[ P_s = \frac{1}{1+\frac{b_1+b_2}{b_1}\sqrt{1-\beta(1-\delta)^2}} \frac{a}{\sqrt{1-\beta(1-\delta)^2}} + \frac{2\beta}{\sqrt{1-\beta(1-\delta)^2}} W. \]

(3.34)

**Proof:** With \( v(D_{it};D_t) \) taking the form (3.31), the dynamic equations (3.29) and (3.30) become

\[ (P_{it} - W) - \beta(1-\delta) (P_{i,t+1} - W) = (D_{it} - (1-\delta)D_{i,t-1}) \left( -b_1 + \beta(1-\delta)p'(D_{it};D_t) \right) \]
\[ a - b_1 D_{it} - b_2 D_t = P_{it} - \beta(1-\delta)P_{i,t+1} \]

Plugging the assumed form of the equilibrium (3.32) and (3.33) into these equations and comparing coefficients of different terms leads to six conditions:

\[ (1-\beta(1-\delta)) \bar{P} = a - (b_1 + b_2) \bar{D} \]
\[ 1 - \beta(1-\delta) \phi_x = (b_1 + b_2) \frac{\phi_x}{\bar{D}} \]
\[ 1 - \beta(1-\delta) \phi_a = b_1 \frac{\phi_a}{\bar{P}} \]
\[ \frac{\phi_a}{\phi_x} \delta \bar{D} = (1 - \beta(1-\delta)) (\bar{P} - W) \]
\[ -\frac{\phi_a}{\phi_x} \left( 1 - \frac{1-\delta}{\phi_x} \right) = 1 - \beta(1-\delta) \phi_x \]
\[ -\left( 1 - \frac{1-\delta}{\phi_x} \right) = 1 - \beta(1-\delta) \phi_a \]
Note that here we used $p' (D_{t-1}; D_{t-1}) = -\alpha$, implied by (3.33). Algebraic manipulations lead the result (3.34). It is straightforward to verify that with these values all equilibrium conditions are satisfied. ■

**Result 14** The collective persistence parameter is smaller than the idiosyncratic persistence parameter: $\phi_k < \phi_a$.

**Proof:** The values of these parameters given in Lemma 13 may be rewritten as

$$\phi_k = \frac{1 - \delta}{1 + \frac{b_1 b_2}{b_1 + b_2} \sqrt{1 - \beta (1 - \delta)^2}}, \quad \phi_a = \frac{1 - \delta}{1 + \sqrt{1 - \beta (1 - \delta)^2}}.$$  

Since $(b_1 + b_2)/b_1 > 1$, the inequality $\phi_k < \phi_a$ immediately follows. ■

**Result 15** The price $P_{it}$ of a particular variety of durable goods depends more strongly on the deviations of the average stock of durables $D_{t-1}$ from the steady state value $\bar{D}$ than on the deviations of the stock $D_{it-1}$ of this variety from the average $D_{t-1}$: $\kappa > \alpha$.

**Proof:** Start with the inequality

$$\frac{b_1}{b_1 + b_2} \left(1 - \beta (1 - \delta)^2\right) + \sqrt{1 - \beta (1 - \delta)^2} > \frac{1}{1 - \beta (1 - \delta)^2 + \sqrt{1 - \beta (1 - \delta)^2}},$$

which is certainly satisfied due to $(b_1 + b_2)/b_1 > 1$. Multiplying both sides by $(1 - \delta) b_1$ and then manipulation each side separately leads to

$$\frac{(1 - \delta) (b_1 + b_2)}{1 - \beta (1 - \delta)^2 + \frac{b_1 + b_2}{b_1} \sqrt{1 - \beta (1 - \delta)^2}} > \frac{b_1}{\beta (1 - \delta)} \left( \frac{1}{\sqrt{1 - \beta (1 - \delta)^2}} - 1 \right),$$

which is, due to Lemma 13, equivalent to $\kappa > \alpha$. ■
3.6 Conclusion

We evaluate price and quantity dynamics in several environments such as monopoly, oligopoly and monopolistic competition. We show that in all these environments, in response to cost shocks, markups are countercyclical and therefore pass-through is incomplete. We contrast these findings with that of nondurable goods pricing results. We also contrast this to the case of marginal cost pricing of durable goods which is the typical assumption in the macroeconomic literature.

In this work we have limited our study to the pricing problem of firms and therefore the analysis has been partial equilibrium in nature. In future versions we plan to add general equilibrium elements into the model and study the implications of the pricing dynamics of durable goods for aggregate macroeconomic variables both in closed and open economy environments.
In this appendix, we prove part (ii) of Lemma 2, namely $p'(D_{t-1}; W_t, \xi_t) \leq 0$ and $f'(D_{t-1}; W_t, \xi_t) \geq 0$. We first show that the second derivative of the value function cannot be negative. The firm’s Bellman equation is

$$V(D_{t-1}; W_t, \xi_t) = \max_{P_t} \{(P_t - W_t) (d(P_t; \xi_t) - (1 - \delta) D_{t-1}) + \beta \mathbb{E}V(d(P_t; \xi_t); W_{t+1}, \xi_{t+1})\}.$$ 

Denote the value of $P_t$ that the firm actually chooses when the state variable is $D_{t-1}$ by $P_t^{(0)}$. Now consider a small change to the initial stock of durables:

$$V(D_{t-1} + \epsilon; W_t, \xi_t) = \max_{P_t} \{(P_t - W_t) (d(P_t; \xi_t) - (1 - \delta) D_{t-1} - (1 - \delta) \epsilon) + \beta \mathbb{E}V(d(P_t; \xi_t); W_{t+1}, \xi_{t+1})\}.$$ 

If the firm selects a wrong price, its value cannot increase:

$$V(D_{t-1} + \epsilon; W_t, \xi_t) \geq \left\{ \left(P_t^{(0)} - W_t\right) \left(d(P_t^{(0)}; \xi_t) - (1 - \delta) D_{t-1} - (1 - \delta) \epsilon\right) + \beta \mathbb{E}V(d(P_t^{(0)}; \xi_t); W_{t+1}, \xi_{t+1}) \right\},$$

$$V(D_{t-1} + \epsilon; W_t, \xi_t) \geq - \left(P_t^{(0)} - W_t\right) (1 - \delta) \epsilon + V(D_{t-1}; W_t, \xi_t).$$
By replacing \( \varepsilon \to -\varepsilon \) we obtain another inequality of this type. Adding the two inequalities gives

\[
\frac{1}{2} V(D_{t-1} + \varepsilon; W_t, \xi_t) - V(D_{t-1}; W_t, \xi_t) + \frac{1}{2} V(D_{t-1} - \varepsilon; W_t, \xi_t) \geq 0.
\]

Taking the limit \( \varepsilon \to 0 \) we get \( V''(D_{t-1}; W_t, \xi_t) \geq 0. \)

The envelope theorem \( V'(D_{t-1}; W_t, \xi_t) = - (1 - \delta) (p(D_{t-1}; W_t, \xi_t) - W_t) \) implies

\[
V''(D_{t-1}; W_t, \xi_t) = - (1 - \delta) p'(D_{t-1}; W_t, \xi_t),
\]

which means that \( p'(D_{t-1}; W_t, \xi_t) \leq 0. \) Given this result, and the assumption of concave utility function, (3.7) implies \( d'(p; W_t, \xi_t) \leq 0. \) Differentiating the definition of the function \( f \) gives

\[
f'(D_{t-1}; W_t, \xi_t) = d'(p(D_{t-1}; W_t, \xi_t); W_t, \xi_t) p'(D_{t-1}; W_t, \xi_t; W_t, \xi_t).
\]

We conclude that \( f'(D_{t-1}; W_t, \xi_t) \geq 0. \)


Fabinger, M., and E. G. Weyl (2012): “Incidence, demand forms and imperfect competition,” This work is in progress.


